



Optimal design of drainage networks in Kut city using genetic algorithms and spectral analysis of the river to know the impact of network pollutants

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Abstract

The essential objective of this study is the development of an appropriate model to obtain the low cost optimization design of the sewage network. The complexity and the huge number of discrete and non-linear constraints in problems of sewage system design make their treatment important. For this aim, an adaptive model of the Genetic Algorithm (GA) of efficient and effective optimization design with a consistent layout is proposed. The MATLAB code was used to optimize the sewage network for the Al Jafriya neighborhood and Al Kut commercial center. Spectral analysis of the Tigris River was also carried out for the area of the estuaries of the studied network to find out the extent of pollution due to the network being a joint network (rain and sewage). According to the obtained results, the developed model achieved the optimum solution with the minimum cost and least generations number. It can be also noticed that the estuaries of the studied storm system have a significant influence on the quality of the river water. Thus, this research proposes the implementation and management of an isolated sewage system that is routed to wastewater treatment plants.

Keywords: Optimal design, image classification, supervised classification, ArcGIS

الخلاصة: الهدف الأساسي من هذه الدراسة هو تطوير نموذج مناسب للحصول على التصميم الأمثل منخفض التكلفة لشبكة الصرف الصحي. إن التعقيد والعدد الهائل من القيود المنفصلة وغير الخطية في مشاكل تصميم نظام الصرف الصحي يجعل معالجتها مهمة. لهذا الهدف، تم اقتراح نموذج تكيفي للخوارزمية الجينية (GA) لتصميم التحسين الفعال مع تخطيط منسق. تم استخدام كود MATLAB لتحسين شبكة الصرف الصحي لحي الجفرية ومركز الكوت التجاري. كما تم إجراء التحليل الطيفي لنهر دجلة لمنطقة مصبات الأنهار في الشبكة المدروسة لمعرفة مدى التلوث الناتج عن الشبكة كونها شبكة مشتركة (الأمطار والصرف الصحي). ووفقاً للنتائج التي تم الحصول عليها فإن النموذج المطور حقق الحل الأمثل بأقل تكلفة وأقل عدد من الأجيال. يمكن ملاحظة أن مصبات الأنهار لنظام العاصفة المدروس لها تأثير كبير على جودة مياه النهر. وبالتالي، يقترح هذا البحث تنفيذ وإدارة نظام الصرف الصحي المعزول الذي يتم توجيهه إلى محطات معالجة مياه الصرف الصحي.

1. INTRODUCTION

The sewage system can be considered is an essential side of the complicated ecosystem of a city. Sewage systems are form the most significant infrastructures that work to discharge sewage and rainwater. Several regions around the world have separate rainwater and sewage systems. The double connection leads to an increase in pollution as a result of the illegal connection of sewage to rainwater networks only, as these networks drain into the river causing pollution to the river, as well as linking networks of car oil shops and factories, and the water raised by these places. And the pollution it contains, which leads to a threat to the aquatic environment. and the life of the population in general. The significance of this type of systems is to maintain people safety and to keep a healthful aquatic environment that does not risk aquatic life and wildlife. The danger lies in the dual system as it increases the pollution of the river, which affects the life of the inhabitants as well as the aquatic life [1].

One of the most important solutions to the problem of networks, especially the old ones, which were designed for conditions that differ from what is the case at the present time, such as unimplemented spaces and others, is to re-design them. One of the design requirements is that it meets the economic requirements and achieves the lowest

cost [2, 3]. This topic has sparked the interest of researchers, who are working to exploit and build cost-effective optimization and design strategies [4]. The term genetic algorithm (GA) can be defined as a searching algorithm depending on natural selection and the population genetics mechanisms [5]. The use of GA for effective search to solve for optimal planning of drainage networks is subject to various influential factors, involving size of population, fitness function, coding type, three essential operators (selection, mating and mutation), and number of generations [6].

Merritt and Bogan [7] used the method of optimal design to add a pump station for the existed sewage system. Froise and Burges [8] used a design method in which the network is improved or designed with the addition of a pumping station. The model of discrete differential dynamic programming (DDDP) was used by Li and Matthew [9] for optimizing the sewage network with the pumping station. Pan and Kao [10] hybridized GA with quadratic programming (QP) to the optimal sewage network design and created pumping and gravity substitutions for the static layout of the sewage network with appropriate computational time. Sewage is illegally released into storm water drainage systems in several urban areas. In this situation, it is important to regulate and manage the aspects related to the sewage networks for all countries. The goal is the providing of appropriate wastewater treatment prior to its direct release into water body, and the implementation of strict laws for constraining the pollution sources.

Techniques of remote sensing helps researchers and decision makers in monitoring the spatial changes without the need for site monitoring, this reduces the effort and cost [11, 12]. Images can be classified by the GIS software according to the information. There are two ways of classification, unsupervised classification (a computer-guided), and supervised classification (user-guided) [13]. The used technique in the current research is supervised classification. Several researchers depended on spectral classification of rivers in their investigations. Al-Kubaisi et al., (2010) [14] depended on the data of three satellites (QuickBird, Spot5, and Landsat7 ETM with a resolution of 0.6, 2.5, 30 m, respectively) for spectroscopy the water of Tigris inside Baghdad by ERDAS software to estimate the contaminants in the river. The obtained results showed that the water can be divided into three types, shallow water, polluted water, and natural water. Allawai and Ahmed (2019) [15] applied ArcGIS program with techniques of remote sensing to spectroscopy Tigris water in Mosul, northern Iraq. Images of Landsat 8 were chosen for analyzing seasonal variations for four seasons depending on method of supervised classification.

The purpose of this research was to create an adaptive strategy for improving the effectiveness of GA formulation and reducing the search span. This research also aims to answer the question: Will the estuary discharge have a substantial influence on the quality of river water? Using ArcGIS Supervised Classification, this article looked at the effect of storm drainage system on the future water body represented by the Tigris.

2. STUDY AREA AND DATA

2.1. Study area

The study area is Al-Jaffryah Quarter and Al-Kut Trade Center located in the city of Al-Kut on the left bank of the Tigris River. The data of stormwater network of the study area were collected from the Department at Wasit Sewerage Directorate. The AutoCAD maps contained information about sub-catchment areas and drainage pumps. Also, the data included information about the manhole depth, invert elevations, types, length, and depth of conduit. Figure (1) Shows the study area.

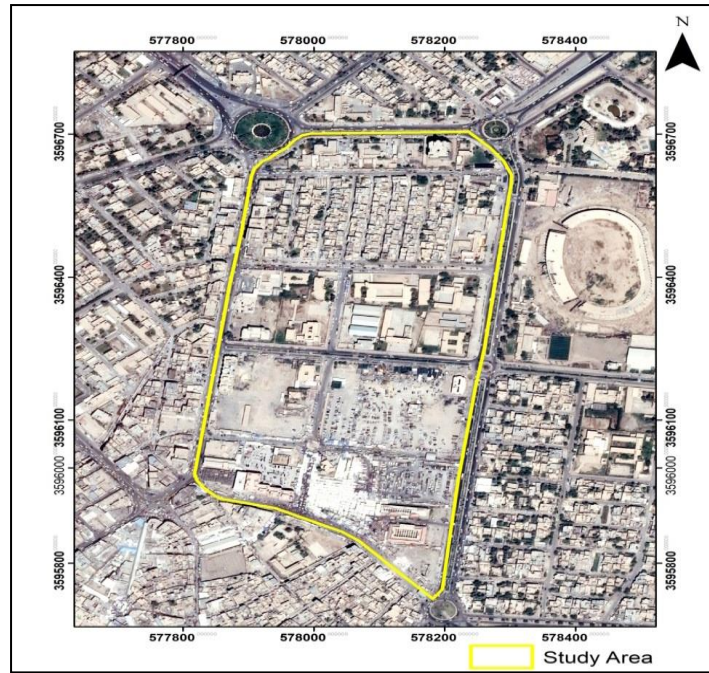


Figure1. The study area location, due to Iraqi map (QuickBird image, 2018).

2.2. Collection of data

The stormwater network schematic of Al-Jaffryah Quarter and Al-Kut Trade Centre was obtained using the high resolution satellite image, stormwater network data from Waist Sewerage Directorate, and field work by using AutoCad 2018 and SWMM 5.1 model. The rain networks in this study contain various types of pipes and other accessories such as manholes (MH), road groove (RG) (the number of network pipelines is about 93 and Manhole 92, respectively with a larger number of road grooves). Pipe diameters range from From 315 mm to 700 mm UPVC secondary lines of 200 mm diameters used for connection between (MH) and (RG). The pumping station in the study area contains a collection basin and three centrifugal pumps produced by Caprari Pump (UK) Ltd. of rain water starting from the surface Ground to (RG) and then to manholes, collected and transported through underground network pipes to the pumping station storage basin, and then pumped to the downstream point on the Tigris River. Distance between manholes ranged from 20 to 70 meters and depths ranged from 2.0 to 3.5 meters. The distance between the manholes or the nodes depends on the lengths of the streets in the first place and according to the design map allocated to the data of the concerned department (the Water and Sanitation Department in Kut), and the standard distances for these lengths are from 20 to 70. The values of pipe prices, manholes, and commercial prices were also obtained, as the prices of manholes vary according to their type and the depth used for each type. As for the price of pipes, it varies according to the diameters of each pipe per unit length. Finally we got the price for installing the pump and the complete station. Its cost was calculated directly from the competent authority and is estimated at thirty million Iraqi dinars. The installation of the complete station, which contains three pumps, with the cost of building and installing them. Tables (1) and (2) show the required prices for each of the pipes and manholes according to their types.

Table 1: Prices of manholes according to their types and the depth used for each type in Iraqi dinars (Kut Sewerage Directorate)

| Manhole | Depth (m) | Total cost (*10 ³) |
|---------|-----------------|--------------------------------|
| Type AS | Not exceed 1.69 | 1250 |
| Type BS | 1.7-3 | 2300 |
| Type BD | More than 3 | 3000 |
| Type CS | 1.7-3.24 | 5000 |
| Type CD | More than 3.25 | 5200 |

Table 2: The price of each pipe according to the diameter in Iraqi dinars (Kut Sewerage Directorate).

| Diameter of pipe (m) | Total cost (*10 ³) | Unit |
|----------------------|--------------------------------|------------|
| 0.25 | 80 | unit meter |
| 0.315 | 100 | unit meter |
| 0.4 | 150 | unit meter |
| 0.5 | 210 | unit meter |
| 0.6 | 300 | unit meter |
| 0.7 | 375 | unit meter |

The images of Quickbird satellite related to Tigris in the investigation area was purchased from an external bureau. Image of US QuickBird satellite was used with a spatial resolution of 25 cm, date of acquisition was on December 15, 2019.

3. METHODOLOGY

3.1. Optimum Design of Sewer Network

One of the solutions to the issue of networks, especially the old ones is to re-design them. One of the design requirements is that it meets the economic requirements. In order to obtain the optimal design from an economic point of view and to achieve technical standards, mathematical programming techniques are used, and one of these techniques is the genetic programming method.

3.1.1. Genetic Algorithm

Holland [16] was the first who used the term genetic algorithm (GA), it is an algorithm of searching depended on the natural selection and the population genetics mechanism. Holland [16] applied genetic algorithm in water distribution pipeline systems. The GA can be considered one of the newest methods of optimization for the designing of sewage networks, it works iteration by iteration successively generating and testing a population of strings.

- Formulation of optimization

The construction cost of a sewage network can be estimated by the following equation. It was used in a recent study in Karbala for the same purpose It is a recent study and it was relied upon because it is inside Iraq and for the same conditions and country, where the networks are almost similar in all governorates in terms of pipe diameters and other conditions such as drilling depths. With dependence on the prices for the city of Kut, which were obtained as mentioned previously.[17]

$$Min C = \sum_{l=1}^{Np} l_l K_p (D_l Z_l^i Z_l^j) + \sum_{i=1}^{Np+1} K_m hm_i + \sum_{i=1}^{Nn} K_{pp} Q_i \quad (1)$$

where, C represents the overall cost of the sewer system (units), Np represents number of network pipes, Nn indicates the number of network nodes, l_l represents the length of l^{th} pipes, D_l is the diameter of l^{th} pipes, $Z_l^i Z_l^j$ is the downstream and upstream excavation depth of l^{th} pipe, Kp is the unit cost of construction related to every pipe, it is a function of its diameter (D_l), hm_i represents the manhole's depth, Km indicates the unit construction cost of i^{th} manhole, Q_i represents the discharge, and Kpp represents the coefficient of cost for the pump installing at the i^{th} node. Table (3) It shows the special equations for calculating the cost for each pipe and manhole according to the conditions [17].

Table 3: Cost components of construction for the case study in (1000 ID /m)

| For pipe, Kp | |
|---|---|
| $(-5490 D_i^2 + 3182.6 D_i - 5.03 Z_i + 71.43 Z_i D_i - 378) (482.3$ $D_i^2 + 2.565 D_i + 14.7 Z_i + 19.06 Z_i D_i + 46.35)$ $(1792.14 D_i^2 - 3639.2 D_i - 30.5 Z_i + 226.03 Z_i D_i + 2268.03)$ | $20 \text{ cm} \leq D_i \leq 315 \text{ cm}$ $100 \text{ cm} \leq Z_i \leq 600 \text{ cm}$ $40 \text{ cm} \leq D_i \leq 90 \text{ cm}$ $100 \text{ cm} \leq Z_i \leq 800 \text{ cm}$ $100 \text{ cm} \leq D_i \leq 180 \text{ cm}$ $400 \text{ cm} \leq Z_i \leq 800 \text{ cm}$ |
| For manholes, Km | |
| $(-24.3 Z_m^2 + 1411.7 Z_m + 22.9 DM - 6.74 Z_m DM - 1830.2)$ | For all depths and sizes |

- Constraints

When thinking about designing a network dedicated to a specific area or thinking about redesigning it, the first step of the design is to set parameters that must be adhered to, and the results we get must be within these limits. And so on until a suggested pipe diameter with a suitable discharge velocity is obtained between the upper and lower limits of velocity. The speed depends on the diameter of the tube used.

The following points represent the constraints of the model:

- 1) The velocity of flow must remain between the maximum and minimum limit to prohibit sedimentation and scouring, and to provide the self-clean ability, respectively.

$$V_{min} \leq V_i \leq V_{max} \quad i = 1, \dots, NP$$

Where, V_i is the peak flow velocity in i^{th} pipe, V_{min} is the lowest acceptable peak flow velocity, and V_{max} is the highest acceptable velocity.

- 2) The outlet pipe's diameter (D_l') must be specified similar to or greater than upstream inlet pipes (D_l), for every manhole.

$$D_l \leq D_l'$$

- 3) The selected diameters of the pipes must be existed in the commercial pipes list.

$$D_l \in D_{commercial}$$

- 4) The discharge must be in the range between the maximum and minimum limits depending on the value of diameter of pipe and velocity.

- Basic GA

1. [Start] creates random population of (n) chromosomes (appropriate solutions for a problem).
2. [Fitness] evaluates the fitness $f(x)$ for every chromosome (x) of the population.
3. [New population] a new population is generated by repeating the following steps till it is created:
 - a) [Selection] two parent chromosomes are selected from the population due to their fitness (higher fitness means higher chance to be chosen).

- b) [Crossover] the parents is passed to create a fresh children (offspring). When there is no passing to be performed, offspring will be a perfect copy of parents.
 - c) [Mutation] mutates new offspring at every locus (location in chromosome).
 - d) [Accepting] locates the new children in a new population.
4. [Replace] uses the lately generated population for a further algorithm run.
 5. [Test] when satisfying the end condition, stop, and the best solution must be returned in current population.
 6. [Loop] start form the second step.

- Selection

It is a process of selecting two native populations for reproduction during an evolutionary operation. It can be concluded from the title that this method includes creating a number of offspring proportional to the individual fitness. In the present study, two methods were used: One-Point Crossover (OPC)), and Roulette wheel selection (RWS).

3.2. Classification of satellite image

According to the evaluation of many researchers, the technique of supervised classification can be considered the most appropriate method for spectral classification by technique of remote sensing [18]. The algorithm of maximum likelihood in supervised classification can be considered the best large-scale approach in the analysis of satellite image data, it shows a high degree of classification accuracy. In the current investigation, the technique of supervised spectral classification depending on ArcGIS program was used for determining the types of water existed in the investigated part of Tigris. The analyst closely controls supervised classification. During this process, pixels, which represent land cover features or patterns, are recognized, or they can be determined from another source, such as maps, data of ground truth, or aerial photos. The desired classes and the collection of the data are required before classification [19].

4. RESULTS AND DISCUSSION

4.1. Results of a genetic algorithm model for the sewer network

The code for improvement was made based on a recent study that was carried out in Karbala Governorate. The local prices of the Wasit Sewers Directorate were relied upon, as well as the depths of special drilling when implementing the network (Atiyah et al. 2021). The results of the Roulette wheel selection (RWS), and the One-Point Crossover (OPC) were used as the best of many methods in the relationship among the optimal design of the diameters of sewer pipes. New diameters for the case study grid were designed using a minimum diameter of 250 mm because most international standards use this diameter, as well as some Iraqi research published in Iraq, the researchers used a minimum diameter of 250 mm or 200 mm as the minimum commercial diameter that can be widely used networks in the country. The total cost of the network was reduced from (1,664 million Iraqi dinars manual design to (1,479 million Iraqi dinars optimal design with minimum diameter = 250 mm) which led to a decrease of about (10%). The comparison includes excavations, pipes, and manholes. The best improvement was obtained after about 73 generations. In this study, first, the diameters of the pipe are optimized, the drainage of each pipe is optimized, and then the speed is improved. Keeping the true slope as the appropriate direction of the design and maintaining both upstream and downstream, that is, an improvement was made to the tubes only in terms of their diameters, then the discharge and velocity of each tube. These designs were obtained using the tournament selection method, one-point crossover, crossover probability (P_c) = 1, one gene mutation per chromosome, mutation probability (P_m) = 0.05. The comparison includes excavations, pipes, and manholes. The optimization was terminated after about 73 generations.

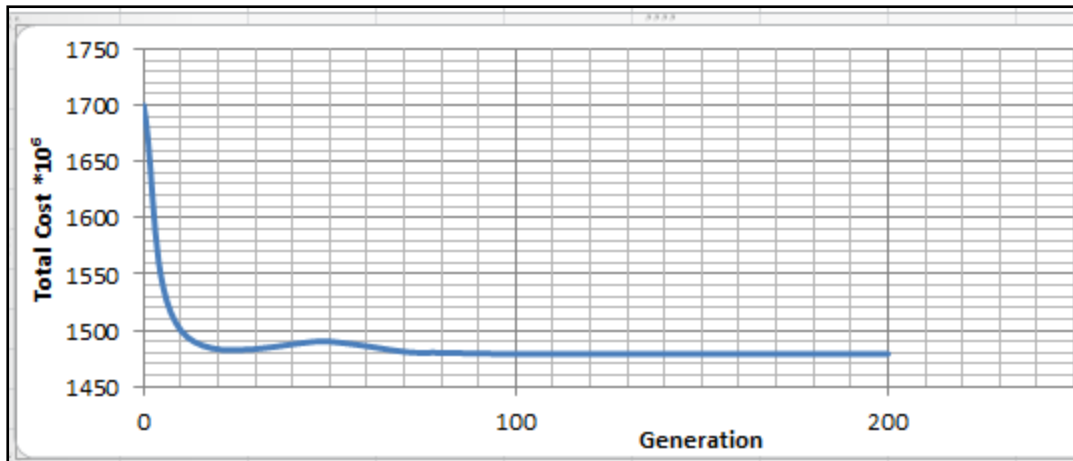


Figure 2: Lowest cost optimized

The optimization was done by keeping the true slope as the appropriate direction of the design and maintaining both upstream and downstream, i.e. tubes were optimized only in terms of their diameters, then the discharge and velocity of each tube. The properties of the optimum design using the current model are listed in Table 3. The actual data of the pipes or the truth was entered, that is, their real diameters were entered, according to the prices mentioned in Table No. 2, and the new diameters were obtained after the generation of 73 generations of new diameters mentioned in Table No. 4, according to the prices. The final cost was calculated after the improvement, which amounted to 1,479 million Iraqi dinars optimal design with a minimum diameter = of 250 mm

Table 4: The properties of optimum design using the current model.

| Pipe Dimeter(m) | Optimum Dimeter(m) | Optimal Discharge (m^3/s) |
|-----------------|--------------------|-------------------------------|
| 0.315 | 0.25 | 0.02 |
| 0.4 | 0.315 | 0.03 |
| 0.5 | 0.4 | 0.07 |
| 0.6 | 0.5 | 0.13 |
| 0.6 | 0.6 | 0.21 |
| 0.7 | 0.7 | 0.32 |

4.2. Analysis of satellite imagery and representation of pollutants

Supervised maximum likelihood classification can be considered the most appropriate large-scale method in the analysis of satellite image data and the most widely used by researchers, which is characterized by high classification accuracy [20]. This type of classification was used in this study to clarify the spectral classification of Tigris in the investigated region by the satellite image data of Quickbird. Four categories of river water can be noticed, polluted water, pure water, shallow area or sediment, and vegetation as depicted in Figure 3.

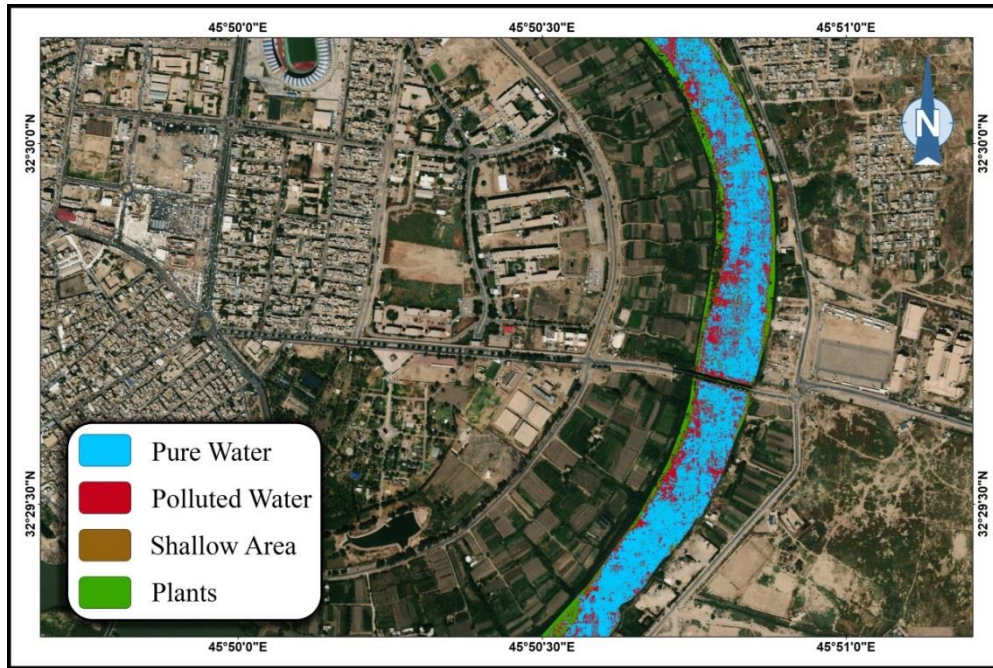


Figure 3: Results of the process of supervised spectral classification.

Sewage pollutants Pollutants from commercial stores and shops selling car oils and other pollutants found in residential areas. The dark blue in the figure refers to the pure waters, the red indicates the polluted water, the light green color refers to the shallow water or sediments, and the dark green shows to the plants. A intensity of the red color was observed, which may reach about 3 km from the beginning of the mound and to about 1 km from under the mound, which indicates the presence of a high percentage of pollutants that are transmitted to the river with rainwater and mixed with sewage water. The accumulation of polluted water on the side of the river has also been observed where the flow of water on the side is slower than in the middle of the river, in addition to the fact that the banks of the river have vegetation, which impedes the movement of water and thus the accumulation of pollutants. The area of water classes and its percentages are listed in Table (4). Details of the used satellite image are listed in Table (5).

Table 5: Areas of water classes and their percentages.

| Class of water | Area (Km ²) | Percentage of area (%) |
|----------------|-------------------------|------------------------|
| Polluted | 0.095492 | 22.05 |
| Pure | 0.265005 | 61.51 |
| Shallow Area | 0.039752 | 9.18 |
| Plants | 0.032743 | 7.56 |

Table 6: The pixels of the satellite image

| Class of water | Pixels | Percentage of pixels (%) |
|----------------|---------|--------------------------|
| Polluted | 1071127 | 22.05 |
| Pure | 2972535 | 61.21 |
| Shallow Area | 445897 | 9.18 |
| Plants | 367271 | 7.56 |

It is clear from these results that the percentage of the contaminated surface area on the surface of the river was 22.05%, and here it turns out that it is a high and influential percentage that must be addressed. It is also necessary not to use water from these areas directly and to educate the population living near those areas and alert them about these risks. Figure (5) shows pollution close-up.

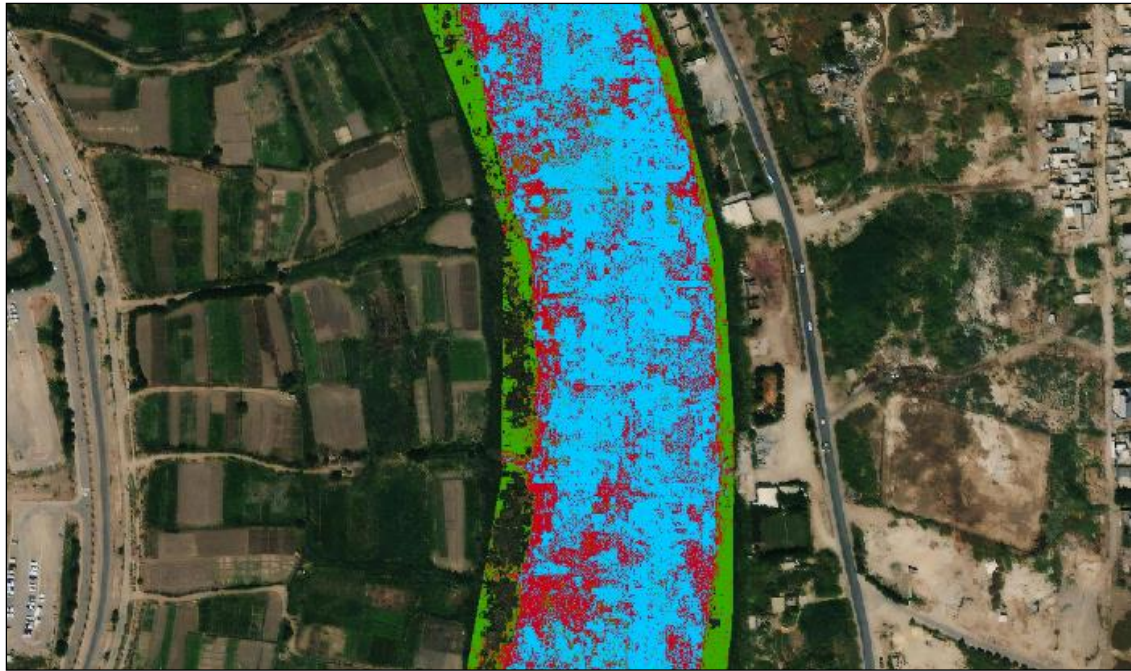


Figure 4: Close-up of the river after classification

5. CONCLUSIONS AND RECOMMENDATIONS

This paper presents a genetic algorithm to improve sewage network systems (pipe diameters and their dependence on drainage and flow velocity). The results were presented and compared with the actual cost. The obtained results indicated that the outfall of the stormwater drainage network that operates as a combined network due to the illegal connection of wastewater had a significant impact on the water quality of the river. Thus, the authors suggest taking the necessary measurements in managing the drainage of this network and implementing a separate sanitation network that would be directed to treatment plants directly. Authors recommended for future researchers to use other satellites with high spatial accuracy to identify polluted water in other drainage networks outfall.

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