NUMERICAL SIMULATION FOR PREDICTION OF WETNESS CONTENT IN WET STEAM FOR CONVERGENT - DIVERGENT NOZZLE

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Abstract:

This research is to find a wetness using the (laser beam) optical Forward Scattering Method (F.S.M.) which is applicable to calculate the wetness in a convergent-divergent De-Laval steam nozzle operating (1.3 Mach number) which may be works at wet steam with pressure (1) bar and temperature (373) K. The light source of He-Ne laser of wave light (λ=0.632) µm was used to prediction the wetness in nozzle a wet steam flow. Both droplet diameter of water (Dr) and relation of intensity of light S= ln(I_B/I_o) are assumed to be (Dr=10,30,50,70,100 µm) and (S = 0.9,.0.8,0.7,0.6,0.5 ) respectively. From the relation of light intensity pattern of many diameter droplet of water (Dr) and different droplet size distribution N (Dr), the MATLAB program can calculate the light attenuation coefficient (Ks) consequently. The increase of the droplet size distribution, N (Dr), leads to decrease the values of (S) and (Ks). The increase of the droplet diameter causes increase of the scattered light, and the minimum value of scattering light is with (Dr= 10µm) for the tested samples. The wetness of steam (y_o) = (1,3,5,7,10) % ,which depend on the [Dr, N(Dr)] in the scattered zone(Vt), can be determined easily by the MATLAB program. The radius of droplet water in two – phase can be adversely calculated by using the research output that then the concentrations or wetness is previously specified.

Key words: Droplet diameter, Wetness, Laser scattering, Mie theory, Supersonic nozzle.
هو ليزر هيليوم- نيون بطول موجي مقداره (0.632) ميكرو متر. ان كل من قطر قطرات بخار الماء 
(\text{Dr})، وبعلاقة شدتي الضوء (S) قد فرضتها وكانت (S = \ln(I_{\beta}/I_o)) = 0.9 و 0.8 و 0.7 و 0.6 و 0.5 على التوالي ومن علاقة شدة الضوء لأقطار مختلفة لقطرات بخار الماء
(\text{Dr}) وتوسيع حجم القطرات (N(\text{Dr}))، تم حساب معامل إضعاف الضوء (K_s) باستخدام برنامج الماتلاب. ان زيادة 
توزيع حجم القطرات يؤدي إلى قلة كل من معامل إضعاف الضوء وعلاقة شدتي الضوء. ان زيادة قطر قطرات 
الماء بسبب زيادة في استطارة الضوء، وإن اقل استطارة في هذا البحث تكون عند قطر قطرات مقداره (10) 
ميكرو متر. وكذلك فإن نسبة الرطوبة (0,1,3,5,7,10) ط. النسبة المئوية في منطقة استطارة 
الضوء يمكن الحصول عليها بسهولة من خلال برنامج الماتلاب. يمكن حساب نصف قطر قطرات بخار الماء في 
التدفق ثنائي الطور باستخدام المعادلة المذكورة (3) لنتائج البحث عندما يكون التركيز أو نسبة الرطوبة معلومة.

\textbf{Nomenclature:}

A= The cross-sectional flow area,\(( \text{m}^2)\)
\beta = The dispersion angle degree.
C= Velocity of light, \((\text{m/s})\).
\text{d}= Distance traveled by the light, \((\text{m})\).
\text{Dr} = Drop diameter, \((\mu\text{m})\).
\text{E}_{\text{photon}} = \text{Photon energy} \,(\text{J})

\text{h}= \text{Planck's constant} \, 6.62620 \times 10^{34} \, \text{Js}
\text{I(\text{d})} = \text{Scattered light intensity}, \,(\text{W/m}^2).
\text{I_o} = \text{Intensity of the light after traveling a distance} \, \text{d}, \,(\text{W/m}^2).
\text{K_s} = \text{Attenuation coefficient might depend on wavelength}, \,(1/\text{m}).

L= The width of nozzle,\,(\text{m}).
M = Mach number.
\text{m} = \text{The mass flow rate.}
\text{m}_s = \text{The mass of steam, kg.}
\text{m}_w = \text{Mass of water in the wet steam kg.}
\text{N(\text{Dr})} = \text{Droplet size distribution.}
\text{N} = \text{Refraction index,} \,(N = 1.33 \text{ for water}) \,.
\text{P_o} = \text{Initial pressure, bar.}
r = \text{laser beam diameter, } \mu\text{m.}
\text{S} = \ln(I_{\beta}/I_o) = \text{relation of intensity of light.}
\text{T_o} = \text{Initial temperature, K.}
\text{V} = \text{Speed in the matter,} \,(\text{m/s}).
\text{V}_{\text{s}} = \text{Scattering zone (volume),} \, \text{m}^3
\text{Y_o} = \text{wetness percentage,} \,\%.
\lambda = \text{Light wavelength,} \,\mu\text{m.}
\text{p}_w = \text{Density of water,} \,\text{kg/} \,\text{m}^3
\text{p}_s = \text{Density of steam,} \,\text{kg/} \,\text{m}^3
\gamma = \text{The} \, \text{ratio of specific heats} .
\text{y} = \text{Drops should be fish optic light.
**Introduction:**

Condensation occurs and droplets grow in the steam nozzle. The droplets cause efficiency reduction of a nozzle. The droplet size calculations are necessary in order to accurately estimate and reduce the mechanical and thermodynamic losses. Where is resulted due to the steam expansion in the convergent-divergent nozzle and specific volume increase for steam, this necessitates the attention to determine the moisture ratio in this case [1]. To know the diameter of water vapor droplets that caused such moisture and to obtain the values of these magnitudes, it is possible to use the optical methods, and among them, that can indicate the dryness fraction in steam nozzle by the MATLAB program of Forward Scattering Method (F.S.M.) Figure (1) [2]. The physical principles of this method is the light propagates rectilinear. This means light propagates in straight lines. In vacuum light travels at a speed $c$ of 300,000 km/s. In matter, light travels less fast. The refractive index $N$ of such matter is defined as the speed in vacuum divided by the speed in the matter $v$:[3]

$$N = \frac{C}{v}$$  \hspace{1cm} (1)

Light is actually electro-magnetic radiation like Rontgen radiation, radio waves, gamma rays, ultraviolet and infrared radiation. Like all other electro-magnetic radiation light consists of waves, with an associated wavelength $\lambda$. Colored light is light of a particular wavelength. In fact white light is a combination of light with different wavelengths. Besides a wave nature, light also exhibits a particle nature: light consists of photons which can be regarded as particles making up the light. A photon represents an amount of light energy $E_{\text{ph}}$, depending on the wavelength of the light.

$$E_{\text{photon}} = \frac{h C}{\lambda}$$  \hspace{1cm} (2)

Light traveling a certain distance through gas, liquid or solid matter is attenuated by absorption and scattering. This attenuation is exponential and is described for coherent light (all light traveling in the same direction) by:

$$I(d) = I_0 \exp (-Ks \cdot L)$$  \hspace{1cm} (3)

**Absorption**

Absorption is the process of transferring of light into thermal energy (heat motion of the molecules). In the absorption, process light disappears. The amount of absorption varies with wavelength of the incident light. In solids and liquids, we see absorption bands: over a range of wavelength, we have continuous absorption, fading off gradually at the ends. In low-pressure gasses, absorption lines are present: light of particular wavelengths (photon energy) is absorbed. [3]
Scattering

Scattering is the process of changing direction of the light. Photons may change direction on collision with molecules of the matter or with particles present in the matter. Mie scattering is scattering due to particles that are larger than the wavelength of the incident light. Mie scattering is not (or hardly) wavelength dependent. Scattered light will look white or light bluish, depending on the size of the scattering particles. Since this scattering is mainly forward directed, it shows to the observer as a halo around the sun. Since there is hardly wavelength dependency, this halo is white. F.S.M. based on the light traveling a certain distance through gas, liquid or solid matter is attenuated by absorption and scattering [2]. Form the applicable side of scientific researches for steam that contains steam with droplets of the two-phase flow, there are many methods to specify the nature of the wet steam: the light extinction method [4, 5] and the side scattering method [6]. In the light extinction method, the transmission of light, need several wavelength. This method has the following disadvantages. If the droplet density is low, I (β) is larger than (0.9) Therefore the variation of I (β) with is very small. In that case droplets largest than (2) µm diameter cannot be calculated if (λ) is in the visible range. The side scattering method the pulse height of light scattered by one particles is single function of (Dr) in certain diameter range. In this method, the scattering zone has to be small so that it contains zero or one particle. The F.S.M. is applicable to droplets at a high density and has a wide calculation rang., droplet larger (0.1)µm diameter can be calculated, and applied in Laval nozzle. However, there is forward scattering method (F.S.M) for the light scattered, in which a light passes through the flow wet steam in the test section at a small angle (β), where the concentration of water droplets through a scattering zone (Vt) with droplet size distribution N(Dr) affect the scattered light, see figure (1). In (F.S.M.), the incident light (Io) of Helium–Neon laser with a wavelength (λ= 0.6328 µm) light crosses the flow wet steam in the nozzle by an angle (€=90°) from the left side, and the detector

![Figure (1) Laser Scattered Light due to the Particles (water droplet) [7].](image-url)
receives the scattering light \( I_B \) at the right side by angle \( \beta = 0^0 \) due to the water droplet in the scattering zone \( V_t \). In this method, the scattered light intensity pattern by many particles is calculated and inverted to droplet size distribution using:

\[
I_B = \int_0^\infty I_o N(Dr) dD \tag{4}
\]

The solution of equation (1) gives the values of the attenuation coefficient \( K_s \) at a assumed diameter of droplet of water \( Dr = 10, 30, 50, 70, 100 \) \( \mu m \) and assumed relation of intensity of light \( S = \ln \left( \frac{I_B}{I_o} \right) = (0.9, 0.8, 0.7, 0.6, 0.5) \). From the fit curves of \( K_s \) and the relative of intensity of light \( S \) then the steam wetness may be obtained with a high accuracy. The quantity of existed steam within that volume is computed, and the volume of droplets presented in the same volume is calculated. Therefore, the wetness within that volume is determined: the steam wetness is determined from the particle size distribution and the physical properties of the wet steam flow [9]. The size of all particles depends on the position and condensation velocity through both high pressure and low pressure in supersonic nozzle [10].

1- FORWARD SCATTERING METHOD

A parallel monochromatic light is directed at many calculated droplets, the angular pattern of the scattered light intensity pattern, is calculated

\[
I_B = \int_0^\infty I_o N(Dr) dD.
\]

\( I_B = \) The scattered light intensity of water droplet, \( W/m^2 \).

\( I_o = \) The intensity of incident light, \( W/m^2 \).

\( N(Dr) = \) Droplet size distribution.

The scattered light \( I_B \) intensity pattern of many droplets can be expressed by equation (1) as superposition of scattered light intensity by one droplet without considering the effect of interference. The calculated droplets are assumed spherical. If the refractive index of droplet water \( N = 1.33 \) and the light source is a He-Ne laser \( \lambda = 0.633 \mu m \). The scattered light is received by detector transforms the scattered light to digit signals.

2- CALCULATION MODEL AND BOUNDARY CONDITION:

A de Laval nozzle (or convergent-divergent nozzle, CD nozzle or con-dinozzle) Figure (2) is a tube that is pinched in the middle, making an hourglass-shape. It is used as a means of accelerating the flow of a gas passing through it to a supersonic speed. It is widely used in some types of steam turbine and is an essential part of the modern rocket engine and supersonic jet engines. Its operation relies on the different properties of gases flowing at subsonic and supersonic speeds. The speed of a subsonic flow of gas will increase if the pipe carrying it narrows because the mass flow rate is constant. The gas flow through a de Laval nozzle is isentropic (gas entropy is nearly constant).
At subsonic flow, the gas is compressible; sound, a small pressure wave, will propagate through it. At the "throat", where the cross sectional area is a minimum, the gas velocity locally becomes sonic (Mach number = 1.0), a condition called choked flow. As the nozzle cross sectional area increases, the gas continues to expand and the gas flow increases to supersonic velocities where a sound wave will not propagate backwards through the gas as viewed in the frame of reference of the nozzle (Mach number > 1.0). A de Laval nozzle will only choke at the throat if the pressure and mass flow through the nozzle is sufficient to reach sonic speeds, otherwise no supersonic flow is achieved.

In addition, the pressure of the gas at the exit of the expansion portion of the exhaust of a nozzle must not be too low. Because pressure cannot travel upstream through the supersonic flow, the exit pressure can be significantly below ambient pressure it exhausts into, but if it is too far below ambient, then the flow will cease to be supersonic, or the flow will separate within the expansion portion of the nozzle, forming an unstable jet that may 'flop' around within the nozzle, possibly damaging it.[3].

\[ \dot{m} = \rho \cdot V \cdot A = \text{constant} \] (5)

Where, \( \dot{m} \) is the mass flow rate, \( \rho \) is the gas density, \( V \) is the gas velocity, and \( A \) is the cross-sectional flow area. If differentiate this equation, obtain:

\[ V \cdot A \cdot d\rho + \rho \cdot A \cdot dV + \rho \cdot V \cdot dA = 0 \] (6)

Divide by \((\rho \cdot V \cdot A)\) to get:

\[ \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \] (7)

Now use the conservation of momentum equation:

\[ \rho \cdot V \cdot dV = -dp \] (8)

And an isentropic flow relation:

\[ \frac{dP}{P} = \gamma \cdot \frac{d\rho}{\rho} \] (9)

Where \( \gamma \) is the ratio of specific heats. The derivation of the isentropic flow relations We can use algebra on this equation to obtain:

\[ dp = \gamma \cdot \frac{P}{\rho \cdot d\rho} \] (10)

and use the equation of state

\[ \frac{P}{\rho} = R \cdot T \] (11)
Where \( R \) is the gas constant and \( T \) is temperature, to get:

\[
dp = \gamma . R \cdot T . d\rho
\]

\( \gamma . R . T \) is the square of the speed of sound \( a \):

\[
dp = (a^2) . d\rho
\]

Combining this equation for the change in pressure with the momentum equation we obtain:

\[
P. V . dV = - (a^2) . d\rho
\]

\[
\frac{V}{(a^2) . dV} = - \frac{d\rho}{\rho}
\]

\[
(M^2) \cdot \frac{dV}{V} = \frac{d\rho}{\rho}
\]

Using the definition of the Mach number \( M = V / a \). Now we substitute this value of \( \frac{d\rho}{\rho} \) into the mass flow equation to get:

\[
(M^2) \cdot \frac{dV}{V} + \frac{dV}{V} + \frac{dA}{A} = 0
\]

\[
(1 - M^2) \cdot \frac{dV}{V} = - \frac{dA}{A}
\]

\[
(M^2) \cdot \frac{dV}{V} = \frac{d\rho}{\rho}
\]

In convergent – divergent nozzle, assume the optical dimensions in the cross section of are width \( L = (50) \) mm Mach number = (1.3), pressure \( P_o = (1) \) bar and the temperature \( T_o = (373) \) K. The shape of the nozzle exhibits the effect of the geometry on the specifications by using various wetness \( (y_o) \) fig (2).

The incident is a visible light of laser Helium–Neon with a wavelength of \( \lambda = 0.6328 \) µm. The incident light is with a diameter of \( r = 0.4 \) mm, the intensity of incident light \( (I_o) \) and the intensity of scattered light \( (I_B) \) depend on the dispersion angle \( (\beta) \). The light attenuation intensity \( (Ks) \), the relation of the intensity of light \( (S = 0.9,0.8,0.7,0.6,0.5) \) and wet steam with various values of The diameter of water drops \( (D_r=10,30,50,70,100) \) µm. That's where electromagnetic waves of light collide with water droplets diameters equal to the wavelength of the light waves are spread and scattered with small angles cause the forward scattering. According to MIE theory of light, the scattering be within the following conditions: 1 - The concentration of water droplets is not large and the ratio of diameter water droplets very small proportion to the distance between the drops' \( \delta = D_r/\varphi<<1 \). 2- Drops should be fish optic lighted \( \Upsilon = 0.3 \).
Figure (2) De Laval nozzle convergent divergent showing the Diagram of approximate flow velocity (v) together with the effect on temperature (T) and pressure (P) in design conditions [9].

Light is passed in first case the test section (nozzle) through a glasses window without steam, and the intensity of the incident light (I₀) is specified in this case, (β=0°), then passing the light again with the existence of wet steam with intensity of the scattering light (I_B) that required to determine its wetness. The intensity of scattered light within different intensities is recorded, depending on the angle (β).

3-THE RELATIONSHIP BETWEEN THE SCATTERING LIGHT AND THE DIAMETER OF DROPLETS AND WETNESS:

The intensity of the scattering light from droplet water calculated by the equations [2];

\[ I_B = I_0 \exp(-K_s \cdot L) \]

(20)

\[ \ln \frac{I_B}{I_0} = -K_s \cdot L \]

(21)

The light attenuation coefficient Ks of the incident light is calculated by the equation;

\[ K_s = \frac{\ln I_0 - \ln I_B}{\pi L \cdot N(Dr)(Dr^2)} \]

(22)

It is possible to determine the water droplet size distribution in the scattering zone by the following equation;

\[ N(Dr) = \frac{3 \cdot m_w}{4 \cdot \pi \cdot p_w \cdot Dr^3} \]

(23)
The scattering zone is calculated from the following formula:

\[ V_t = \frac{1}{2} r^2 \pi L \]  \hspace{1cm} (24)

The mass of water droplet \( m_w \), the mass of steam \( m_s \) and the wetness \( y_o \) can be determined by the following formulas:

\[ m_w = \frac{y_o V_s}{1-y_o} \cdot \rho_s \]  \hspace{1cm} (25)

\[ m_s = V_t \cdot \rho_s \]  \hspace{1cm} (26)

\[ \rho_s = \frac{\rho}{RT} \]  \hspace{1cm} (27)

\[ V_s \approx V_t = volume \ of \ steam \]

\[ y_o = \frac{m_w}{m_s + m_w} \]  \hspace{1cm} (28)

**RESULTS:**

New this research is to use optical methods to determine the percentage of moisture in the steam stream without direct contact with the steam flowing through convergent-divergent nozzle. The F.S.M. is applicable to droplets at a high density and has a wide calculation range., diameter of droplet of water \( Dr = (0.1-300) \mu m \). Curves can also be used that were obtained the diameter of water droplets in two phase flow. Can use the method as a basic of optical sensors in this field.

The calculation of values of the light attenuation coefficient \( K_s \) with the values of relative light intensity \( S \), at diameter of water droplet \( Dr = (10) \mu m \) with wetness \( y_o = (1, 3, 5, 7, 10) \% \), are shown in figure (3).show the value of \( K_s \) at \( S = (0.9) \) and wetness \( y_o=1\% \) is more than that value of \( K_s \) at \( S = (0.5) \) at \( y_o=1\% \). This is as a result of the difference in the N \( (Dr) \) of droplet concentration at \( Dr = (10\mu m) \) in the scattering zone. Thus, one can conclude that the \( (K_s, S) \) curve can be used to predict the wetness due to the droplet concentrations.

From the figure (4) showing the relation between \( (K_s) \) and \( (S) \) at \( Dr = (30) \mu m \), the values of \( (K_s) \), and \( (S) \) at \( y_o= 10\% \) are larger than the values of the wetness \( y_o= (1,3, 5, 7, ) \% \) at the same value of \( Dr \). That means the relative of scattered light at \( y_o=10\% \) is more than in the \( y_o= (1,3, 5, 7, ) \% \) due to the mass of droplet of water in this wetness. Therefore, it can be concluded that the increase of \( (Dr) \) is due to mass increase of water droplet.

The increase of diameter of water droplet to \( Dr = (50) \mu m \) led to increase the mass of water droplet in flow wet steam, and that inverted to increase the values of wetness from \( y_o=1\%, \ y_o=3\%, \ y_o=5\%, \ y_o=7\%, \ y_o=10\% \) respectively, leading to increase the light attenuation coefficient\( (K_s) \), as shown figure (5).

The increase of wetness for \( y_o= (1-10) \% \) at \( Dr = (70) \mu m \) caused an increase in the relative of the scattered light of incident light of laser in the flow wet steam inside the convergent–divergent nozzle, then the values of attenuation coefficient \( K_s \) become more than the values at \( Dr = (10,30,50,) \mu m \), figure (6).

Figure (7) shows the maximum values of \( (K_s) \) with values of \( S= (0.9, 0.8, 0.7, 0.6, 0.5) \) at a diameter of water droplet \( Dr = (100) \mu m \) with the maximum wetness \( y_o=10\% \).
The difference between these values and the values of $K_s$ with $y_o=1\%$ is very large, because the relative of scattered light in the $y_o=1\%$ is less than in the $y_o=10\%$.

**CONCLUSIONS:**

- The increase of the droplet size distribution, $N(\text{Dr})$, leads to decrease the values of $(S)$ and $(K_s)$.
- The increase of the moisture content of $y=(1-10)\%$ when the diameter of the droplets (Dr) const and relationship of light (S) fixed leads to increase the light attenuation coefficient $(K_s)$.
- The increase of the droplet diameter causes increase of the scattered light, and the minimum value of scattering light is with $(\text{Dr}=10\mu m)$ for the tested samples.
- The increase of $(\text{Dr})$ in the flow wet steam in a convergent – divergent nozzle leads to increase the wetness in the flow of two – phase steam.

**REFERENCES:**